OCR Maths M2

Topic Questions from Papers

Collisions

Answers

1	(i)	5m = mu + 4m	M1		cons. of mom.	
		u = 1	A1			
		e = (2-1)/5	M1			
		e = 🗐	A1	4		
	(ii)	I = 4m	B1			
		→	B1	2	to the right	
	(iii)	4m = 5mv	M1			
		v = 😐	A1			
		😐 < 1	B1	3		**9**

(Q4, June 2005)

2	(i)	6m = 3mx + 2my	M1		- 3mx ok if clear on diagram	
		6 = 3x + 2y	A1		m must have been cancelled	
		e = 1 = (y-x)/2	M1		**or** ½.3m.2^2 = ½.3mx^2 + ½.2my^2	
			A1		6 = 3x^2 /2 + y^2 aef	
		x = 0.4 or 2/5	A1		sc A1A0 if x = 2, y = 0 not rejected	
		y = 2.4 or 12/5	A1	6		
	(ii)	4.8m or 24m/5	B1√		√ 2m x their y or 3m(2-their x)	
		same as original dir. of A	B1	2	use their diagram(or dir. of B)	
	(iii)	e = (2.8 – 1.0)/2.4	M1			
		0.75 watch out for ± fiddles	A1√	2	√(1.8/their y) with 0 ⩽ e ⩽ 1	**10**

(Q5, Jan 2006)

3		v^2 = 2gh	M1		kinematics or energy	**2**
		u=√4g or √39.2 or 6.26	A1		speed of impact (±)	
		v=√2.8g or √27.44 (5.24)	A1		speed of rebound (±)	
		I =? 0.3(6.26 + 5.24)	M1		must be sum of mags. of vels.	
		3.45 Ns	A1√	5	√ must be positive	**5**

(Q2, June 2006)

4	(i)	10 = 4 + m.x	M1		conservation of momentum	
		e = … or rationale for x = 2	M1			
		m = 3	A1	3		
	(ii)	v = 6	B1			
		e = 4/5 or 0.8	M1		allow sign errors for M mark	
			A1	3	watch out for lost minuses	
	(iii)	10 – 5 = 2x + y (5 = -2a + b)	M1			
		(-5 = 2c + d)	A1			
		e = 0.8 = (y-x)/10	M1		look for consistency	
		y = x + 8 (a + b = 8) (c – d = 8)	A1			
		x = -1 (a=1) (c=1)	A1		or 1 in opp. direction to 1st	
		y = 7 (b=7) (d=-7)	A1			
		½.2.5^2 +½.1.5^2 - ½.2.1^2 - ½.1.7^2	M1		K.E. lost. Must be 4 parts	
		12 J	A1	8	(37.5 – 25.5)	**14**

(Q8, June 2006)

5		e = 1 = $(y–x)/4$	B1		or ½x0.2x^2 + ½x0.1y^2 =	
		0.8 = 0.2x + 0.1y	B1		½x0.2x4^2(B1/B1 for any 2)	
		solving sim. equ.	M1		not if poor quad. soln.	
		x = 4/3 only	A1	4		**4**

(Q2, Jan 2007)

6	(i)	$x^2 = 21^2$ + 2x40x9.8	M1			
		x = 35	A1			
		$0 = y^2$ – 2x40x9.8	M1			
		y = 28	A1		may be implied	
		e = 28/35	M1			
		e = 0.8	A1	6	aef	
	(ii)	0.2x28 – – 0.2x35	M1		must be double negative	
		I = 12.6	A1	2		**8**

(Q3, Jan 2007)

7 (i)	1.8 = –0.3 + 3m	M1		
	m = 0.7	A1 **2**	**AG**	
(ii)	e = 4/6	M1	accept 2/6 for M1	
	2/3	A1 **2**	accept 0.67	
(iii)	± 3f	B1		
	1/3 ☝ f (⊘ 1)	B1 **2**		
(iv)	I = 3f x 0.7 – – 3 x 0.7	M1	ok for only one minus sign for M1	
		A1		
	I = 2.1 (f + 1)	A1 **3**	aef 2 marks only for -2.1(f + 1)	
(v)	0.3 + 6.3/4 = 0.3a + 0.7b	M1	can be – 0.7b	
	3a + 7b = 18.75	A1 *	aef	
	2/3 = (a – b)/ 5/4	M1	allow e=3/4 or their e for M1	
	3a – 3b = 5/2	A1 *	aef * means dependent.	
	solve	M1		
	a = 2.5	A1	(2.46) allow ± (59/24)	
	b = 1.6	A1 **7**	(1.625) allow ± (13/8)	**16**

(Q7, June 2007)

8 (i)	12 x cos55° 6.88 m s^{-1}	M1 A1 **2**	
(ii)	12 x cos55° x 0.65 (±) 4.47 m s^{-1} ✓	M1 A1 **2**	✓ 0.65 x their (i) **4**

(Q1, Jan 2008)

9 (i)	2mu – 3kmu = –mu + kmv v = ….. v = 3u(1 – k)/k (0 <) k < 1	M1 M1 A1 A1 **4**	 attempting to make v the subject 3u/k – 3u not ≤ 1
(ii)	I = mu – – 2mu 3mu	M1 A1 **2**	or km(3u/k - 3u + 3u) + only
(iii)	v = ± 3u e = (u/2 + 3u)/4u e = 7/8 or 0.875	B1 M1 A1 **3**	 **9**

(Q5, Jan 2008)

10(i)	$u = 3$ m s^{-1}	B1	
	$6 = 2x + 3y$	M1	
		A1	
	$e = (y - x)/3$	M1	
		A1	$(e = ⅔)$ (equs must be consistent)
	$y = 2$	A1 **6**	**AG**

(Q7, June 2008)

11 (i)	$p = 4$ m s^{-1}	B1	P's first speed
	$0.8 = 0.2p_1 + 0.3q_1$	M1	
		A1	
	$0.5 = (q_1 - p_1)/4$	M1	
		A1	
	solving above	M1	
	$q_1 = 2.4$ 12/5	A1	Q's first speed
	$p_1 = 0.4$ 2/5	A1 **8**	may be in (ii). **SR** 1 for both negative
(ii)	$0.8 = 0.2p_2 + 0.3q_2$	M1	
		A1	
	$0.5 = (p_2 - q_2)/2$	M1	
		A1	
	solving above	M1	
	$p_2 = 2.2$ 11/5	A1	
	$q_2 = 1.2$ 6/5	A1 **7**	
(iii)	$R = 0.3 \times 1.2^2/0.4$	M1	
	$R = 1.08$ N	A1 **2**	**17**

(Q7, Jan 2009)

12(i)	$I = 0.9 = 6 \times 0.2 - v \times 0.2$	M1	needs to be mass 0.2
		A1	
	$v = 1.5$	A1 **3**	
(ii)	$0.6 = (c - b)/6$	M1	restitution (allow 1.5 for M1)
		A1	
	$6 \times 0.2 = 0.2b + 0.1c$	M1	momentum (allow 1.5 for M1)
		A1	
	$b = 2.8$	A1	
	$0.4 \times 5 + 0.2 \times 1.5 = 0.4a + 0.2 \times 6$ or $I = 0.9 = -0.4a - -0.4 \times 5$	M1	1st collision (needs their 1.5 for M1)
		A1	
	$a = 2.75$	A1	
	$2.75 < 2.8$	M1	compare v's of A and B (calculated)
	no further collision	A1 **10**	**13**

(Q6, June 2009)

13 (i)	$v^2 = 2\times9.8\times3$ or $2\times9.8\times1.8$	M1	Kinematics or energy
	$v_1 = \sqrt{6g}$ or $\sqrt{58.8}$ or $\frac{7}{5}\sqrt{30}$ or 7.67	A1	Speed of impact (±)
	$v_2 = \sqrt{3.6g}$ or $\sqrt{35.28}$ or $\frac{21}{5}\sqrt{2}$ or 5.94	A1	Speed of rebound (±)
	$I = \pm 0.2(5.94 + 7.67)$	M1	
	2.72	A1ft **[5]**	+ve, ft on v_1 and v_2
(ii)	e = 5.94/7.67	M1	
	0.775 or $\frac{\sqrt{15}}{5}$	A1ft **[2]**	Allow 0.774, ft on v_1 and v_2 **7**

(Q2, Jan 2010)

14	$16 - 12 = 2x + 3y$	M1	
	$4 = 2x + 3y$	A1	aef
	$\frac{1}{2}.2(8)^2 + \frac{1}{2}.3(4)^2$ or $\frac{1}{2}.2x^2 + \frac{1}{2}..3y^2$ or $\pm\frac{1}{2}.2(8^2 - x^2)$ or $\pm\frac{1}{2}.3(4^2 - y^2)$	B1	
	$\frac{1}{2}.2(8)^2 + \frac{1}{2}.3(4)^2 - \frac{1}{2}.2x^2 - \frac{1}{2}...3y^2 = 81$	M1	
	$2x^2 + 3y^2 = 14$	A1	aef
	Attempt to eliminate x or y from a linear and a quadratic equation	M1	
	$15y^2 - 24y - 12 = 0$ or $10x^2 - 16x - 26 = 0$	A1	aef
	Attempt to solve a three term quadratic	M1	
	$x = -1$ (or $x = 2.6$)	A1	
	$y = 2$ (or $y = -2/5$)	A1	
	$x = -1$ and $y = 2$ only	A1	
	speeds 1, 2 away from each other	A1 **[12]**	**12**

(Q5, Jan 2010)

15 (i)	$2mu = 2mv + 3mv$	M1	Conservation of momentum
	$v = 2/5\ u$	A1 A1 **3**	Must be $v =$
(ii)	$e = (3v - v) / u$	M1	Using restitution
	$e = 4/5$	A1 **2**	**AG**
(iii)	Initial K.E. $= 9mv^2 / 2 = 18mu^2/25$	B1 FT	FT on their v from (i)
	Final K.E. $= 9mv^2 / 8 = 9mu^2/50$	B1 FT	FT on their v from (i)
	$\frac{1}{2}\ m\ (V)^2$ = Final K.E.	M1	
	$V = 3\ u\ /5$	A1 **4**	**AG**
(iv)	$4mu / 5 - 3mu/5 = 2mx + my$	M1	Conservation of momentum
	$u / 5 = 2x + y$	A1 FT	FT on their v from (i); aef
	$e = 4/5 = (y - x) / u$	M1 FT	Using restitution
	$4u = 5y - 5x$	A1	FT on their v from (i); aef
	solving 2 relevant equations	M1	
	$x = -u/5$ $y = 3u/5$	A1	
	$y = 3u/5$	A1	
	away from wall (x) + towards wall (y)	A1 **8**	both **17**

(Q6, June 2010)

16	**(i)**		b + a =1.8e	M1 A1	Uses restitution b - a =1.8e
			0.7b - 0.2a=0.2x1.8	M1 A1	Uses momentum 0.7b + 0.2a=0.2x1.8, signs consistent with first eqn
			b =0.4(1+e) a = 1.4e - 0.4 1.4e - 0.4 > 0.4 + 0.4e e > 0.8	M1 A1 A1 M1 A1 **[9]**	Solves 2 simultaneous equations (eliminate a or b) a = 0.4 - 1.4e Using a>b, correct signs in a essential
	OR	**Last 5 marks**	Using a > b a > 0.72 b > 0.72 1.8e > 0.72 + 0.72 e > 0.8	M1 A1 A1 M1 A1	correct signs in a essential
	OR	**Last 5 marks**	Using a = b to find a or b a (or b) = 0.9e and a (or b) = 0.72 e = 0.8 Convincing argument for correct inequality e > 0.8	M1 A1 A1 M1 A1	
	OR	**Last 5 marks**	a = 1.4e - 0.4 or b =0.4(1+e) Using a > b a > 0.9e or b < 0.9e e > 0.8	M1 A1 M1 A1 A1	Solves 2 simultaneous equations (eliminate a or b) aef or multiples thereof correct signs in a essential aef or multiples thereof
	(ii)		c – (±0.25) = 1x0.75 c = 0.5, 1 0.75x0.7 = 0.25x0.7 + m (x1) *OR* 0.75x0.7 = -0.25x0.7 + 0.5m m = 0.35 (from first equation) m=1.4 (from second equation)	M1 A1A1 M1 A1 A1 **[6]**	Uses restitution with e = 1, either Or 0.75 ± 0.25 Uses momentum conservation with correct combination of sign and c value *OR* mx(0.75 ± 0.25) ± 0.7x0.25 = 0.75x0.7
	OR		½ x0.7x0.75^2 = ½ x0.7X0.25^2 + ½mc^2 0.7x 0.75 = 0.7x(+/-0.25) + mc Solving simultaneous equations m = 0.35 m = 1.4	B1 M1 A1 M1 A1 A1	½ may not be seen At least one momentum equation mc = 0.35 and 0.7

(Q7, Jan 2011)

17 ia	If reversed 2.9 + 2 = e(3 + 1.5) e > 1 impossible	M1 A1 [2]	 Award B1 if no explicit numerical justification
b	2.9 - 2 = e(3 + 1.5) e = 0.2	M1 A1 [2]	May be seen in ia
ii	 3m – 0.2x1.5 = 2m + 0.2x2.9 m = 0.88	M1 A1 A1 [3]	Conservation of momentum Accept with g included consistently Do not award if g used
iii	0.68 = 0.2v + 0.2x2.9 v = 0.5 e = 0.5/2.9 e = 0.172	M1 A1 M1 A1 [4]	Impulse = change in momentum Separation speed not 2.9 Allow 5/29

(Q4, June 2011)

18	(i)		$v^2 = 2 \times 9.8 \times 3.136$	M1	Uses $v^2 = u^2 + 2as$ or energy with $u = 0$. Signs must be consistent.
			$v = 7.84$	A1	
			Rebound speed $= 7.84e$	B1 FT	Ignore –ve.
			$I = \pm 0.5(7.84 + 7.84e) = \pm 3.92(1 + e)$	B1 FT [4]	AEF seen. FT on cv(v).
	(ii)		$-7.84e = 7.84e - gt$	M1	Uses a complete method to find t.
			$t = 1.6e$ **AG**	A1 [2]	
	(iii)	(a)	$t_2 = 1.6e^2$	B1	
		(b)	$t_3 = 1.6e^3$	B1 [2]	
	(iv)		Time to first bounce is 0.8 s	B1	
			Identify total time is sum of a GP in e	B1	Indication of the sum of at least to term in e^4
				M1	Equate 3.4 or 4.2 or 5 or 5.8 with attempt at use of formula for sum to infinity of a GP.
			$\frac{1.6e}{1-e} = 4.2$	A1	
			$e = 0.724$	A1 [5]	Allow 21/29

(Q6, Jan 2012)

19	(i)		Speed $= 1.2$ ms^{-1}	B1	May be seen anywhere, even in **(ii)**; allow -1.2
			Impulse $= 0.8 \times \pm (4 - -1.2)$	M1	Difference between momenta, allow $0.8 \times \pm (4 - 1.2)$
			± 4.16 Ns	A1 **[3]**	
	(ii)		KE lost $= ½ \times 0.8 \times (4^2 - (\pm 1.2)^2)$	M1	
			5.82(4) J	A1 **[2]**	Allow -5.82(4)

(Q1, June 2012)

20	(i)		$0.2 \times 1.8 = 0.2v_A + 0.4v_B$	*M1 A1	Attempt at conservation of momentum
			$v_B - v_A = ⅓ \times 1.8$	*M1	Attempt at restitution
			Solve for v_A or v_B	A1 Dep*M1	aef
			$v_B = 0.8$ m s^{-1} and $v_A = 0.2$ m s^{-1} **AG**	A1 **[6]**	
	(ii)		$0.4 \times 0.8 + 0.6 \times 0.2 = 0.4v_{B'} + 0.6v_C$	B1	
			$v_C - v_{B'} = e(0.8 - 0.2)$	B1	aef
			Use two relevant equations to eliminate v_C	*M1	
			State $v_{B'} \geq 0.2$	B1	soi, Allow $v_{B'} > 0.2$
			Set up (in)equality in e and their v_A	dep*M1	Condone incorrect inequality sign for M1 only
			$0.44 - 0.36e \geq 0.2$ or $0.44 - 0.36e = 0.2$	A1	Allow $0.44 - 0.36e > 0.2$
			$e \leq 2/3$ or 0.667	A1 **[7]**	
		OR	$0.4 \times 0.8 + 0.6 \times 0.2 = 0.4v_{B'} + 0.6v_C$	B1	
			$v_C - v_{B'} = e(0.8 - 0.2)$	B1	aef
			State $v_{B'} \geq 0.2$	B1	soi, Allow $v_{B'} > 0.2$
			Sub $v_{B'}$ in momentum equation & solve for v_C	*M1	
			($v_C =$) 0.6	A1	
			Set up (in)equality in e and their v_A	dep*M1	eg $0.6 - e(0.8 - 0.2) \geq 0.2$, Condone incorrect inequality sign for M1 only
			$e \leq 2/3$ or 0.667	A1 **[7]**	

(Q6, June 2012)

21	**(i)**		a = gsin30 1+u = 0.4(2+2gsin30) u = 3.72 ms^{-1}	B1 M1 A1 A1 **[4]**	Using NEL with u_A from cv(a), u_A ≠ 0 cwo	
	(ii)		Use $v^2 = u^2 - 2(\text{gsin30})s$ s = 1.41 m	M1 A1 **[2]**	Using v = 0, cv(a) from (i) or correct a SC If a not found in (i), allow a=g for M1A0.	
	(iii)		Use of conservation of momentum 0.5 x 2gsin30 - 2m = m - 0.5 x 3.72 m = 2.25	M1 A1ft A1 **[3]**	Using cv(a) ft cv(u) from (i) Aef(raction) eg 2 $^{19}/_{75}$ or $^{169}/_{75}$	

(Q3, Jan 2013)

22		**(i)**		M1	Use of restitution, may have sign errors, must be correct ratio (v/u)
			$4 - 4(1 - e + e^2) = -e(u - 4)$	A1	
			$u = 4e$	A1	oe
				M1	Use of conservation of momentum
			$mu + 0.2 \times 4 = 0.2 \times 4(1 - e + e^2) + 4m$	A1	
			$m = 0.2e$	A1	oe
				[6]	
		(ii)	Valid method to find e that gives the least speed	M1	Differentiate v_A and equate to 0 or complete the square on v_A
			Get e = ½	A1	www
				M1	Difference of KE with 4 terms
			½ × 0.2 × 4^2 + ½ × 0.1 × 2^2 – (½ × 0.2 × 3^2 + ½ × 0.1 × 4^2)	A1	Must have found the value of e from a legitimate method. www
			(+/–) 0.1 J	A1	SCM1A1 Loss of KE = $8e(1 - e)^3/5$ or $8e(1 - 3e + 3e^2 - e^3)/5$ or $8e/5 - 24e^2/5 + 24e^3/5 - 8e^4/5$
				[5]	
		(iii)		*M1	Attempt to use impulse = change in momentum on one particle
			$0.2e(4 - 4e) = 0.192$ or $0.2(4 - (4 - 4e + 4e^2)) = 0.192$	A1	
			Solve three term QE in e	dep*M1	method should lead to 2 real values for e
			e = 0.4 or 0.6	A1	For both
				[4]	

(Q6, June 2013)